PAPER REFERENCE: T200-5

Development of a General Purpose Code to Couple Experimental Modal Analysis and Damage Identification Algorithms

Scott W. Doebling[1], Charles R. Farrar[1], Phillip J. Cornwell[2]

[1]Engineering Analysis Group, Los Alamos National Laboratory
Los Alamos, NM, 87545, USA

[2]Dept. of Mechanical Engineering, Rose Hulman Institute of Technology
5500 Wabash Ave.
Terre Haute, IN, 47805, USA

ABSTRACT

This paper introduces a new toolbox of graphical-interface software algorithms for the numerical simulation of vibration tests, analysis of modal data, finite element model correlation, and the comparison of both linear and nonlinear damage identification techniques. This toolbox is unique because it contains several different vibration-based damage identification algorithms, categorized as those which use only measured response and sensor location information ("non-model-based" techniques) and those which use finite element model correlation ("model-based" techniques). Another unique feature of this toolbox is the wide range of algorithms for experimental modal analysis. The toolbox also contains a unique capability that utilizes the measured coherence functions and Monte Carlo analysis to perform statistical uncertainty analysis on the modal parameters and damage identification results.

INTRODUCTION

This paper introduces a new suite of graphical-interface software algorithms to numerically simulate vibration tests and to apply various modal analysis, damage identification, and finite element model refinement techniques to measured or simulated modal vibration data. This toolbox is known as DIAMOND (Damage Identification And MOdal aNalysis of Data), and has been developed at Los Alamos National Laboratory over the last year. DIAMOND is written in MATLAB [1], a numerical matrix math application which is available on all major computer platforms. The development of DIAMOND was motivated primarily by the lack of graphical implementation of modern damage identification and finite element model refinement algorithms. DIAMOND is unique in three primary ways:

1. DIAMOND contains several of the most widely used modal curve-fitting algorithms. Thus the user may analyze the data using more than one technique and compare the results directly. This modal identification capability is coupled with a numerical test-simulation capability that allows the user to directly explore the effects of various test conditions on the identified modal parameters.

2. The damage identification and finite element model refinement modules are graphically interactive, so the operation is intuitive and the results are displayed visually as well as numerically. This feature allows the user to easily interpret the results in terms of structural damage.

3. DIAMOND has statistical analysis capability built into all three major analysis modules: modal analysis, damage identification, and finite element model refinement. The statistical analysis capability allows the user to determine the magnitude of the uncertainties associated with the results. No other software package for modal analysis or damage identification has this capability.

OVERVIEW OF DIAMOND

DIAMOND is divided into four primary modules at the top level: numerical vibration test simulator, experimental modal curve fitting and statistical analysis, damage identification, and finite element model refinement. These four modules constitute the primary hierarchy in DIAMOND, as shown in Figure 1. In this paper,

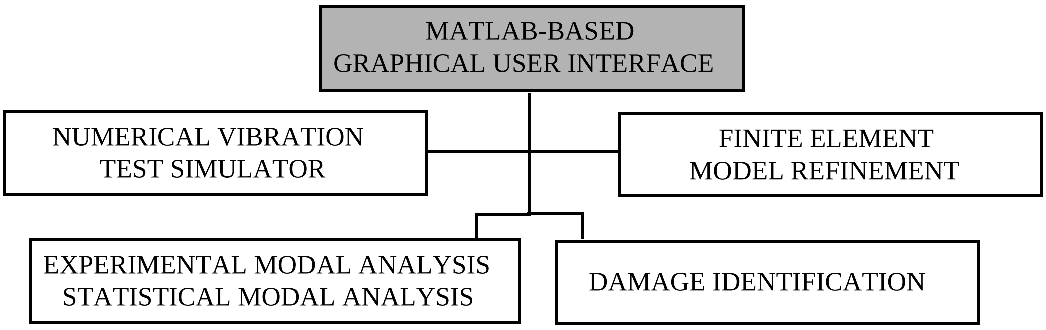

Figure 1: Flowchart of Top Level of DIAMOND

the three analysis-oriented modules (all but the test simulator) will be discussed in further detail.

Experimental Modal Analysis / Statistical Analysis of Modal Data

The experimental modal analysis module provides a series of tools for plotting the data in various forms, plotting data indicator functions, defining sensor geometry, performing modal curve-fits, analyzing the results of modal curve fits, and analyzing the variance of identified modal parameters as a function of the noise in the measurements as defined by the measured coherence function. A flowchart of this module is shown in Figure 2. The most important feature of the experimental modal analysis module is the variety of modal parameter identification algorithms which are available. These include: Operating shapes, the Eigensystem Realization Algorithm (ERA), [2], Polyreference Time Domain [3], the Rational Polynomial Curve fit [4], and the Levenberg-Marquardt nonlinear least-squares curve fitting routine [5]. Any of these modal identification algorithms can be implemented in a statistical Monte Carlo [6] technique. In such an analysis, a series of perturbed data sets, based on the statistics of the measured FRFs as defined by the measured coherence functions, are generated and propagated through the selected algorithm. The statistics on the results are then used as uncertainty bounds on the identified modal parameters.

Damage Identification

The algorithms contained in the damage identification module of DIAMOND can be classified as modal-based, finite element refinement-based, or nonlinear. A flowchart of the damage identification module is shown in Figure 3. The damage identification module presents a number of different algorithms:

1. Strain energy methods are based on the work of Stubbs [7], Cornwell [8], and others. The basic idea of these methods is the division of the structure into a series of beam or plate-like elements, and then the estimation of the strain energy stored in each element both before and after damage. The curvatures (second-derivatives with respect to space) of the mode shapes are used to approximate the strain energy content.

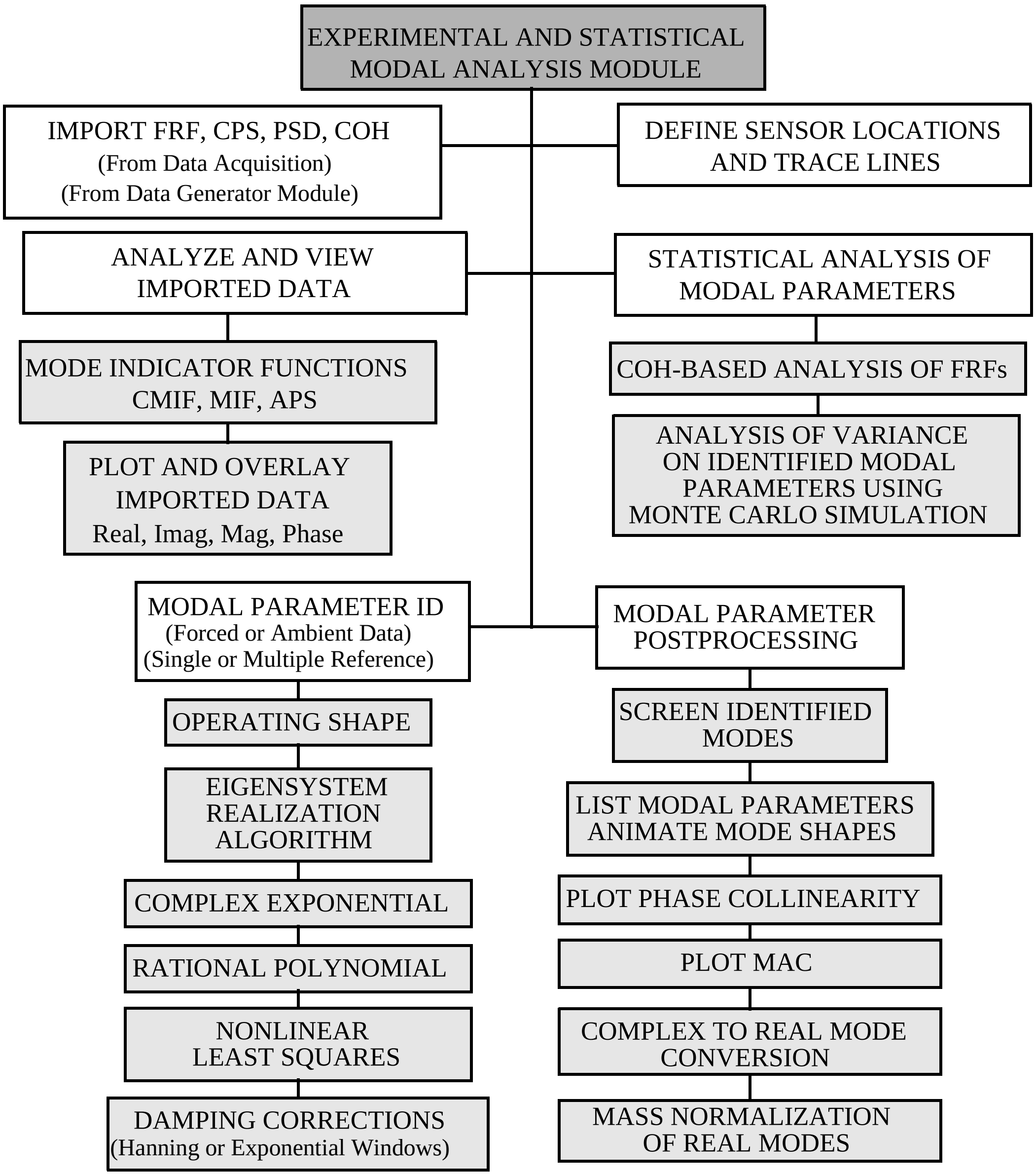

Figure 2: Flowchart of Experimental Modal Analysis / Statistical Modal Analysis Module

2. Flexibility methods all use some measure of the change in the modal flexibility matrix. The modal flexibility matrix is used to estimate the static displacements that the structure would undergo as a result of a specified loading pattern.[9] The flexibility matrix is estimated from the mass-normalized measured mode shapes, $[\Phi]$, and modal frequencies squared, $[\Lambda]$, as

$$[G] \approx [\Phi][\Lambda]^{-1}[\Phi]^T \tag{1}$$

3. Finite element model correlation-based damage identification techniques are based on the comparison of the finite element model correlation results from before damage to those after damage. The correlation techniques are discussed in the next section.

4. Nonlinear damage identification techniques are based on different theories of nonlinear signal processing. They are a widely varying group of methods and are reviewed and discussed in Ref. [10].

Finite Element Model Refinement

The finite element model refinement module consists of four options: pre-processing for update analysis, optimal matrix updating, sensitivity-based model update, and post-processing of update results. A flowchart of

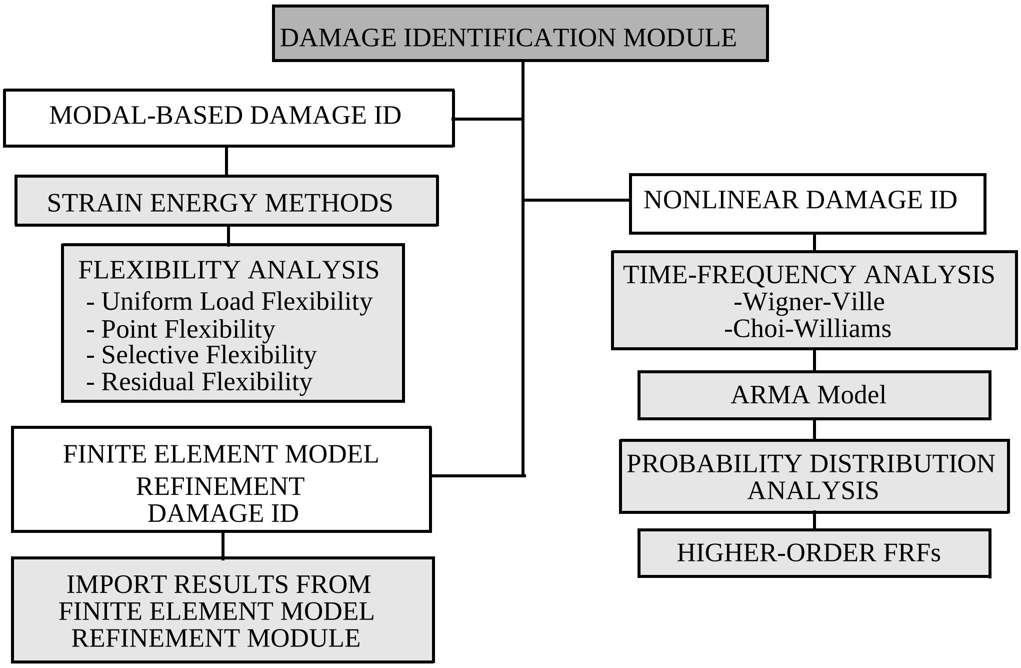

Figure 3: Flowchart of Damage Identification Module

this module is shown in Figure 4.

The pre-processing phase of the model correlation analysis involves the selection of which modal parameters (i.e. modal frequencies and mode shapes) should be used in the correlation, as well as which finite element model parameters should be updated.

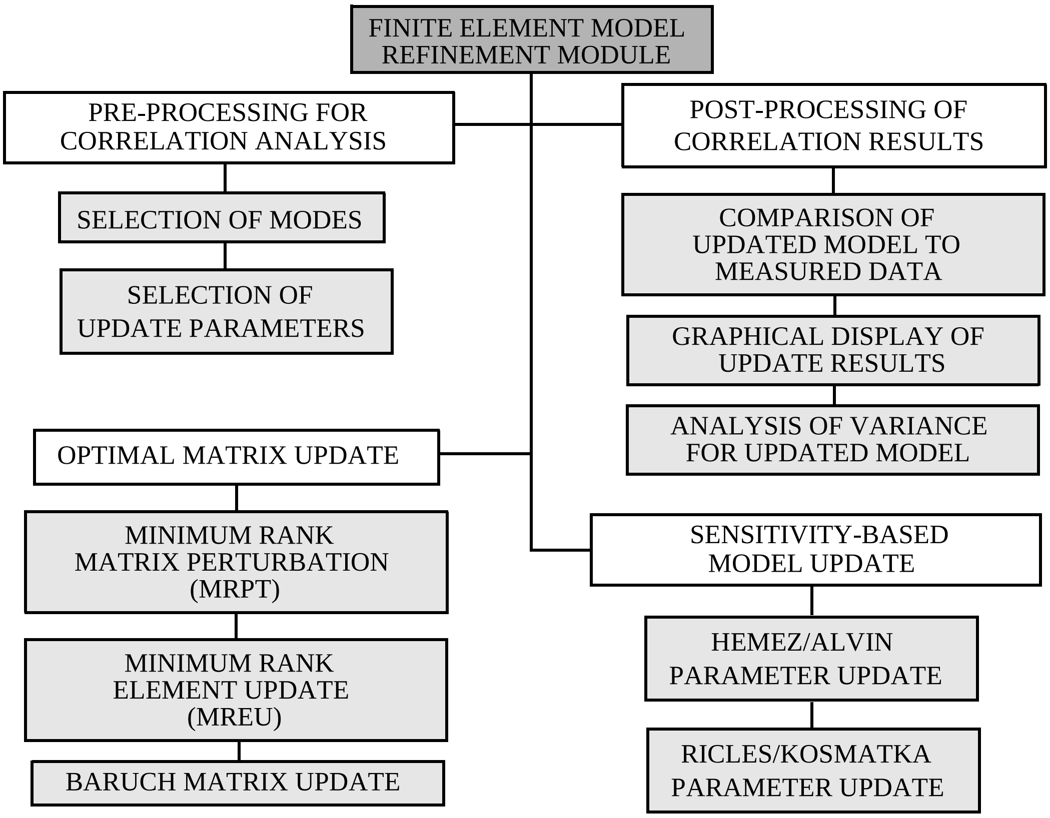

Figure 4: Flowchart of Finite Element Model Refinement Module

The optimal matrix update methods are based on the minimization of the error in the structural eigenproblem using a closed-form, direct solution. The minimum rank perturbation technique (MRPT) [11] is one such method which produces a minimum-rank perturbation of the structural stiffness, damping, and/or mass matrices reduced to the measurement degrees of freedom. The minimum rank element update (MREU) [12] is a similar technique which produces perturbations at the elemental, rather than the matrix, level. The Baruch updating technique [13] minimizes an error function of the eigenequation using a closed-form function of the mass and stiffness matrices.

The sensitivity-based model update methods also seek to minimize the error in the structural eigenequation, but do so using a Newton-Raphson-type technique based on solving for the perturbations such that the gradient of the error function is near zero. [6] Thus these methods require the computation of the sensitivity of the structural eigenproblem to the parameters which are to be updated. The Hemez/Alvin algorithm [14],[15] computes the sensitivities at the elemental level, then assembles them to produce the global sensitivity matrices. The Ricles/Kosmatka [16] algorithm computes a "hybrid" sensitivity matrix using both analytical and experimental sensitivities.

EXAMPLE APPLICATION: THE ALAMOSA CANYON BRIDGE

The following analysis of modal data from a series of tests performed on a highway bridge is intended to demonstrate the application of the unique capabilities of DIAMOND to data from an actual field test.

The Alamosa Canyon Bridge has seven independent spans with a common pier between successive spans. An elevation view of the bridge is shown in Figure 5. The bridge is located on a seldom-used frontage road

Figure 5: Elevation View of the Alamosa Canyon Bridge

parallel to Interstate 25 about 10 miles North of the town of Truth or Consequences, New Mexico, USA. Each span consists of a concrete deck supported by six W30x116 steel girders. The roadway in each span is approximately 7.3 m (24 ft.) wide and 15.2 (50 ft.) long. Between adjacent beams are four sets of channel section cross braces equally spaced along the length of the span. At the pier the beams rest on rollers, and at the abutment the beams are bolted to a half-roller to approximate a pinned connection.

The data acquisition system was set up to measure acceleration and force time histories and to calculate frequency response functions (FRFs), power spectral densities (PSDs), cross-power spectra and coherence functions. Thirty averages were typically used for these estimates. A total of 31 acceleration measurements were made on the concrete deck and on the girders below the bridge as shown in Figure 6. Two excitations points were located on the top of the concrete deck. A modal sledge hammer was used as the impact excitation source. Point 2 was used as the primary excitation location. Point 23 was used to perform a reciprocity check. More details regarding the instrumentation can be found in Ref. [17].

A total of 52 data sets were collected over the course of the six days of testing. Temperature measurements were made at 9 locations around the bridge, both above and below the deck, to track the effects of ambient temperature changes on the test results. Reciprocity and linearity checks were conducted first. A series of modal tests was conducted over a 24 hour period (one test every 2 hours) to assess the change in modal properties as a result of variations in ambient environmental conditions, as discussed in Ref. [17]. Damage cases presented in this paper are results from simulated stiffness reduction using a correlated FEM. The first three identified modal frequencies and mode shapes from the modal analysis of one of these data sets are shown in Figure 7.

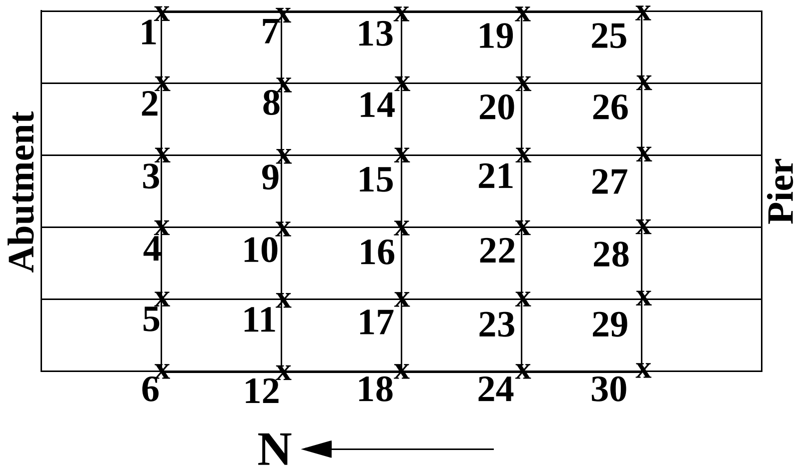

Figure 6: Accelerometer and Impact Locations

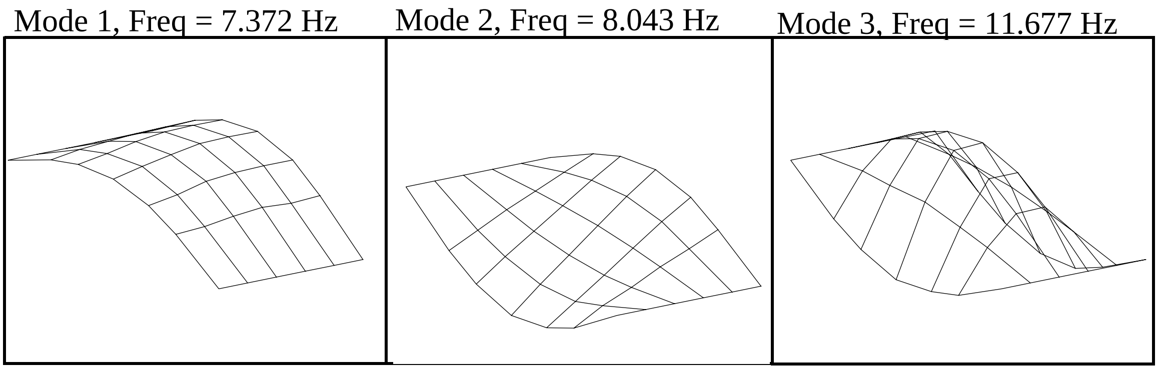

Figure 7: Identified Mode Shapes for Alamosa Canyon Bridge

Analysis of Uncertainty in Each Test

Statistical uncertainty bounds on the measured frequency response function magnitude and phase were computed from the measured coherence functions, assuming that the errors were distributed in a Gaussian manner, according to the following formulas from Bendat and Piersol [18]:

$$\begin{aligned} \sigma(|H(\omega)|) &= \frac{\sqrt{1-\gamma^2(\omega)}}{|\gamma(\omega)|\sqrt{2n_d}}|H(\omega)| \\ \sigma(\angle H(\omega)) &= \frac{\sqrt{1-\gamma^2(\omega)}}{|\gamma(\omega)|\sqrt{2n_d}}\angle H(\omega) \end{aligned} \tag{2}$$

where $|H(\omega)|$ and $\angle H(\omega)$ are the magnitude and phase angle of the measured FRF, respectively, $\gamma^2(\omega)$ is the coherence function, n_d is the number of measurement averages, and $\sigma(\bullet)$ is the value of 1 standard deviation (68% uncertainty bound). These uncertainty bounds represent a statistical distribution of the FRF based on a realistic level of random noise on the measurement. Once the 1 standard deviation (68% uncertainty) bounds were known, 2 standard deviation (95% uncertainty) bounds were computed. Statistical uncertainty bounds on the identified modal parameters (frequencies, damping ratios, and mode shapes) were estimated using the uncertainty bounds on the FRFs via a Monte Carlo analysis [6]. The details of this procedure are shown in Ref. [19].

Effects of Uncertainties on Damage Identification

The changes in the bridge as a result of damage were predicted using the finite element model. The damage case modeled was the 100% failure of a connection between a cross-member and an interior girder. A comparison of the estimated 95% confidence bounds and the predicted changes as a result of damage for the modal frequencies are shown in Figure 8. The modal frequencies of modes 3, 4, 7, 8, and 9 undergo a change that is

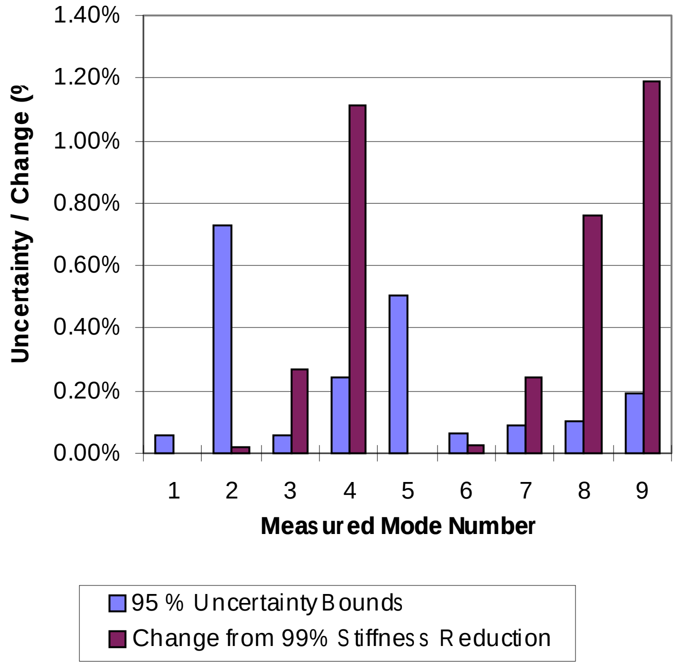

Figure 8: Comparison of Modal Frequency 95% Confidence Bounds to Changes Predicted as a Result of Damage

significantly larger than the corresponding 95% confidence bounds. The relative magnitudes of the changes indicate that the frequency changes of these modes could be used with confidence in a damage identification analysis. It should be noted from the y-axis scale of Figure 8 that the overall changes in frequency as a result of damage are quite small (< 1.2%), but as a consequence of the extremely low uncertainty bounds on the modal frequencies (many less than 0.2%), these small changes can be considered to be statistically significant.

The relative statistical significance of the changes in the various modes is one of the primary motivating factors for a “selective flexibility” approach. Using the selective flexibility approach, those modes where the frequency changes are statistically insignificant would be considered to be unchanged, while those modes with significant frequency change would be used in the flexibility analysis. This technique would prevent modes which are insensitive to the damage from masking the indications of damage from modes which are sensitive to the damage.

CONCLUSION

A new toolbox of graphical-interface software algorithms, known as DIAMOND, has been introduced and demonstrated. The toolbox provides the capability to simulate vibration tests, perform experimental modal analysis including statistical bounds, apply various damage identification techniques, and implement finite element refinement algorithms. The structure of the toolbox menus was described in detail in this paper, and a sample application to measured data from a highway bridge was presented.

ACKNOWLEDGMENTS

This work was supported by Los Alamos National Laboratory Directed Research and Development Project #95002, under the auspices of the United States Department of Energy. The authors wish to recognize the contributions of Mr. Erik G. Straser and Mr. A. Alex Barron of Stanford University.

REFERENCES

1. MATLAB, *Users Manual*, The Mathworks, Inc., 1994.

2. Juang, J.N. and Pappa, R.S., “An Eigensystem Realization Algorithm for Modal Parameter Identifica-

tion and Model Reduction," *Journal of Guidance, Control and Dynamics,* Vol. 8, No. 5, pp. 620-627, 1985.

3. Vold, H., Kundrat, J., Rocklin, T., and Russell, R., "A Multi-Input Modal Estimation Algorithm for Mini-Computers," SAE Transactions, Vol. 91, No. 1, January 1982, pp. 815-821.

4. Richardson, M.H. and Formenti, D.L., "Parameter Estimation from Frequency Response Measurements using Rational Fraction Polynomials," *Structural Measurement Systems Technical Note 85-3,* 1985.

5. McVerry, G.H., "Structural Identification in the Frequency Domain from Earthquake Records," Earthquake Engineering and Structural Dynamics, Vol. 8, 1980, pp.161-180.

6. Press, W.H., Teukolsky, S.A., Vetterling, W.T., and Flannery, B.P., *Numerical Recipes in FORTRAN: The Art of Scientific Computing,* Second Edition, Cambridge Univ. Press, 1992, pp. 684-686.

7. Stubbs, N., J.-T. Kim, and C.R. Farrar, 1995, "Field Verification of a Nondestructive Damage Localization and Severity Estimation Algorithm," in *Proc. 13th International Modal Analysis Conference*, 210–218.

8. Cornwell, P., Doebling, S.W., and Farrar, C.R., "Application of the Strain Energy Damage Detection Method to Plate-Like Structures," to appear in *Proc. of the 15th International Modal Analysis Conference,* Orlando, FL, February, 1997.

9. Robinson, N.A., L.D. Peterson, G.H. James, and S.W. Doebling, 1996, "Damage Detection in Aircraft Structures Using Dynamically Measured Static Flexibility Matrices," in *Proc. of the 14th International Modal Analysis Conference,* 857–865.

10. Doebling, S.W., Farrar, C.R., Prime, M.B., and Shevitz, D.W., "Damage Identification and Health Monitoring of Structural and Mechanical Systems From Changes in Their Vibration Characteristics: A Literature Review," Los Alamos National Laboratory report LA-13070-MS.

11. Zimmerman, D.C. and M. Kaouk, 1994, "Structural Damage Detection Using a Minimum Rank Update Theory," *Journal of Vibration and Acoustics,* **116**, 222–230.

12. Doebling, S.W., "Damage Detection and Model Refinement Using Elemental Stiffness Perturbations with Constrained Connectivity," in *Proc. of the AIAA/ASME/AHS Adaptive Structures Forum,* pp. 360–370, AIAA-96-1307, 1996. Accepted and to appear in *AIAA Journal*.

13. Baruch, M., 1982, "Optimal Correction of Mass and Stiffness Matrices Using Measured Modes," *AIAA Journal,* **20**(11), 1623–1626.

14. Hemez, F.M. and C. Farhat, 1995, "Structural Damage Detection via a Finite Element Model Updating Methodology," *Modal Analysis: The International Journal of Analytical and Experimental Modal Analysis,* **10** (3), 152–166.

15. Alvin, K.F., 1996, "Finite Element Model Update via Bayesian Estimation and Minimization of Dynamic Residuals," in *Proc. of the 14th International Modal Analysis Conference,* Dearborn, MI, February 1996, 561–567.

16. Ricles, J.M. and J.B. Kosmatka, 1992, "Damage Detection in Elastic Structures Using Vibratory Residual Forces and Weighted Sensitivity," *AIAA Journal*, **30**, 2310–2316.

17. Farrar, C.R., Doebling, S.W., Cornwell, P.J., and Straser, E.G., "Variability Of Modal Parameters Measured On The Alamosa Canyon Bridge," in *Proc. of the 15th International Modal Analysis Conference,* Orlando, FL, February 1997. 257-263.

18. Bendat, J.S. and Piersol, A.G., *Engineering Applications of Correlation and Spectral Analysis*, John Wiley and Sons, New York, 1980, p. 274.

19. Doebling, S.W., Farrar, C.R., and Goodman, R.S., "Effects of Measurement Statistics on the Detection of Damage in the Alamosa Canyon Bridge," in *Proc. of the 15th International Modal Analysis Conference*, Orlando, FL, February 1997, 919-929.